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Ward identities, $B \to \rho$ form factors and $|V_{ub}|$

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Abstract: The exclusive FCNC beauty semileptonic decay $B \to \rho$ is studied using Ward identities in a general vector meson dominance framework, predicting vector meson couplings involved. The long distance contributions are discussed which results to obtain form factors and $|V_{ub}|$. A detailed comparison is given with other approaches.

Keywords: Weak Decays, B-Physics, Heavy Quarks Physics, Phenomenological Models.

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Contents

1. Introduction

The semileptonic and radiative decays of heavy mesons play an outstanding role for determination of the parameters of standard model, in particular the quark mixing parameters.

The upcomming and currently operating B factories BaBar at SLAC, Belle at KEK, LHCB at CERN and B-TEV at Fermilab as well as the planned τ-Charm factory CLEO at Cornell make precision tests of Standard Model (SM) and beyound SM ever more promising [1]. Especially, a strigent test on the unitarity of the Cabibo-Kobayashi-Maskawa (CKM) mixing matrix [2] in the SM will be made by these facilities. Accurate analysis of exclusive semileptonic B decays are thus strongly demanded for such precision tests. Beauty decays proceeding through the flavour changing neutral current (FCNC) transition have been identified as a valuable source of information on standard model and its extensions [3] (for a review and complete set of refrences see [4, 5, 6]). One of the physics programs at the B factories is the exclusive B decays induced by FCNC transitions. These decays are forbidden at the tree level in the SM and occurs at the lowest order only through one-loop (penguin) diagrams. The experimental tests of exclusive decays are much easier than those of inclusive ones.

The theoretical understanding of weak decays of hadrons, and the measurements of the corresponding CKM matrix elements are consistently hampered by the presence of long distance QCD effects that are responsible for the binding of quarks into hadrons. These effects are hard to evaluate in a model independent way, and so tend to bring large uncertainities to the theoretical predictions for the weak decay amplitudes. They appear in the calculation of the matrix elements of the weak hamiltonian operators, between the initial and final hadronic states.

The dynamical content of hadronic current matrix elements is described by Lorentz-invariant form factors. The calculation of these form factors requires a non-perturbative treatment and are source of large theoretical uncertainities. Knowledge of these form factors is essential for the description of semileptonic and non-leptonic weak decay processes and in particular for the experimental determination of CKM matrix elements [2]. Many theoretical tools have been applied for the determination of form factors, i.e. quark models, QCD sum rule, and lattice calculations. The machinery of sum rules, lattice calculations has the advantage to be directly based on the QCD lagrangian, but is quite involved. Quark models, on the other hand are less directly connected with the QCD lagrangian, but give a vivid picture of what is going on and allow an easy application to different processes and at quite different kinematical regions. However, models presented so far lacked full relativistic covariance with respect to quark spins, and it is the spin structure which determines the ratio of different form factors we are concerned. The knowledge of these ratios is of paramount importance for the analysis of experimentally measured decays. So, the relations between different form factors that will hold under certain conditions or approximations can then be very useful. One of the motivations of the present work is to make use of Ward Identities to relate various form factors in a model independent way. Such relations will reduce the number of uncertain quantities, and improve the accuracy of the theoretical predictions. Moreover, they may help us understand better the general features of the underlying long distance QCD effects.

As mentioned earlier, the theoretical understanding of exclusive decays is complicated mainly due to the non-perturbative hadronic form factors entered in the long distance nonperturbative contributions. These calculations of hadronic form factors for semileptonic B decays have been investigated by various theoretical approches. Form factors based on lattice calculations [7, 8], and on light cone sum rules [9, 10, 11], currently have uncertainties in the 15% to 20% range. A variety of quark model calculations [12]–[17] exist. A number of other approaches [18, 19], such as dispersive bounds and experimentally constrained models based on Heavy Quark Symmetry (HQS) also exists. In the standard model, the short distance contributions are dominated by the top-quark and long distance contributions by form factors.

The aim of the present work, as mentioned earlier, is to relate the various form factors in a model independent way through Ward identities. This enables us to make a clean separation between non-pole and pole type contributions, the $q^2 \to 0$ behaviour of the former is known in terms of a universal function $\xi_\perp\left(q^2\right) \equiv g_+\left(q^2\right)$ introduced in the large energy effective theory (LEET) of heavy (B) to light (ρ) form factors [20]. The corrections of order $(m/M)^2$, where m is the light meson mass and M is the heavy meson mass, in LEET were obtained in [21]. The LEET approach was first proposed by Dugan and Grinstein [22] and further developed by Aglietti [23].

The residue of the pole is then determined in a self consistent way in terms of $g_+(0)$ or $\xi_\perp(0)$ (a sort of duality). Thus our approach has a predictive power as the residue of the pole will give information about the couplings of $B^*(1^-)$ and $B_A^*(1^+)$ with $B\rho$ channel. The form factors are then determined in terms of $g_+(0)$, $A_0(0)$ (for which much information is already known) and pole masses M_{B^*}, $M_{B_A^*}$ (where M_{B^*} is known and estimates exists for $M_{B_A^*}$), and M'/M (where M' is the radial excitation of M (B^* and B_A^*)). This ratio is 1.14 (1.12) [24].

The information about the hadronic form factors thus obtained are used to calculate the decay rates and the CKM matrix element V_{ub}. Because $|V_{ub}|$, the smallest element in the CKM mixing matrix, plays a crucial role in the examination of the unitarity constraints and the fundamental questions on which the constraints can bear [25]. To extract $|V_{ub}|$ from exclusive channel, the form factors for the channel must be known. The form factor normalization dominates the uncertainty on $|V_{ub}|$. The q^2 dependence of the form factors, which is needed to determine the experimental efficiency, also contribute to the uncertainty, but at much reduced level. There have been two exclusive $|V_{ub}|$ analyses by the CLEO Collaboration: a simulatneous measurement of the $B \to \pi l\bar{\nu}$ and the $B \to \rho l\bar{\nu}$ transitions [26], and second measurement of the $B \to \rho l\bar{\nu}$ rate [27]. The branching fractions obtained were

$$\begin{aligned} \mathcal{B}\left(B^0 \to \rho^- l^+\nu\right) &= \left(2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5\right) \times 10^{-4}\ [26] \\ \mathcal{B}\left(B^0 \to \rho^- l^+\nu\right) &= \left(2.69 \pm 0.41^{+0.35}_{-0.40} \pm 0.5\right) \times 10^{-4}\ [27]\,. \end{aligned}$$

The results of the two analyses are largely statistically independent, and they have been combined

$$|V_{ub}| = \left(3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55\right) \times 10^{-3}\,,$$

where the errors arise from statistical, experimental systematic, and form factor uncertainities, respectively. A recent study by BaBar [28], obtained the following results for the branching ratios and $|V_{ub}|$ and

$$\begin{aligned} \mathcal{B}\left(B^0 \to \rho^- l^+\nu\right) &= (3.29 \pm 0.42 \pm 0.47 \pm 0.60) \times 10^{-4}\,, \\ |V_{ub}| &= \left(3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}\right) \times 10^{-3}\,, \end{aligned}$$

where the uncertainities are statistical, systematic, and theoretical respectively; and CLEO [29],

$$\begin{aligned} \mathcal{B}\left(B^0 \to \rho^- l^+\nu\right) &= \left(2.17 \pm 0.34^{+0.47}_{-0.54} \pm 0.41 \pm 0.01\right) \times 10^{-4}\,, \\ |V_{ub}| &= \left(3.00 \pm 0.21^{+0.29}_{-0.35}{}^{+0.49}_{-0.38} \pm 0.28\right) \times 10^{-3}\,, \end{aligned}$$

where the errors in CLEO analyses are statistical, experimental systematic, theoretical systematicbased on lattice QCD and LCSR uncertainities, and $\rho l\nu$ form factor shape uncertainity, respectively.

The future for the exclusive determinations of $|V_{ub}|$ appears promising. For both lattice and B factories, $B \to \pi l\nu$ appears to be a golden mode for future precise determination of $|V_{ub}|$ while the $B \to \rho l\nu$ mode will be more problematic for high precision because the broad width introduces both experimental and theoretical difficulties.

This paper is organized as follows: In section 2, we give the $B(p) \to V(k,\varepsilon)\, l\bar{\nu}$ current matrix elements. In section 3, we discuss the Ward Identities and develop the relations between form factors which result in to reduce the number of unknown quantities. In section 4, pole contribution of various form factors are discussed and relations among different decay constants with the help of Ward Identities are obtained. In section 5, form factors are obtained which are necesary for the calculation of decay width and branching ratio. In section 6, we discuss the decay distribution in terms of helicity amplitudes and the extraction of $|V_{ub}|$ from the experimental data. In section 7, numerical analyses is made and our predictions regarding various observables are compared with other approaches. Section 8, summarizes our conclusions and predictions.

2. Current matrix elements

We are interested in the exclusive semileptonic decays of pseudoscalar meson (B) into vector meson (ρ). The structure of hadronic current in semileptonic decay must be constructed from the available four-vectors, which are momenta and spin-polarization vectors. The Lorentz-vector or axial vector quantities thus formed have Lorentz invariant coefficients (form factors) that are functions of $q^2 = (p-k)^2$. The polarization vector of the meson $\rho(k,\varepsilon)$ leads to a hadronic current with three form factors (in the limit of zero charged lepton mass).

In the case of $B(p) \to \rho(k,\varepsilon)\, l\bar{\nu}$ decay, there are seven form factors for pseudoscalar to vector transitions, i.e. V, $A_{0,1,2}$, and $F_{1,2,3}$ [4, 5, 12]

$$\langle V(k,\varepsilon)|V^\mu|B(p)\rangle = \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M_B+M_V}\varepsilon^*_\nu k_\alpha p_\beta V\left(q^2\right) \tag{2.1}$$

$$\begin{aligned}\langle V(k,\varepsilon)|A^\mu|B(p)\rangle &= (M_B+M_V)\,\varepsilon^{*\mu}A_1\left(q^2\right) - \frac{\varepsilon^*\cdot q}{M_B+M_V}(k+p)^\mu A_2\left(q^2\right) - \\ &\quad -2M_V\frac{\varepsilon^*\cdot q}{q^2}q^\mu\left[A_3\left(q^2\right)-A_0\left(q^2\right)\right]\end{aligned} \tag{2.2}$$

where $V^\mu = \bar{u}\gamma^\mu b$, $A^\mu = \bar{u}\gamma^\mu\gamma^5 b$, and

$$A_3\left(q^2\right) = \frac{M_B+M_V}{2M_V}A_1\left(q^2\right) - \frac{M_B-M_V}{2M_V}A_2\left(q^2\right) \tag{2.3}$$

with $A_3(0) = A_0(0)$,

$$\langle V(k,\varepsilon)|\bar{u}i\sigma^{\mu\nu}q_\nu b|B(p)\rangle = -i\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu k_\alpha p_\beta F_1\left(q^2\right) \tag{2.4}$$

$$\begin{aligned}\left\langle V(k,\varepsilon)\left|\bar{u}i\sigma^{\mu\nu}\gamma^5 q_\nu b\right|B(p)\right\rangle &= \left[\left(M_B^2-M_V^2\right)\varepsilon^{*\mu} - (\varepsilon^*\cdot q)(k+p)^\mu\right]F_2\left(q^2\right) + \\ &\quad + (\varepsilon^*\cdot q)\left[q^\mu - \frac{q^2}{M_B^2-M_V^2}(k+p)^\mu\right]F_3\left(q^2\right)\end{aligned} \tag{2.5}$$

with $F_1(0) = 2F_2(0)$.

Again the term proportional to q^μ in eqs. (2.2), (2.5) only plays an important role for the case $l=\tau$ and is not relevent for the rate in case of produced lepton pair $l=\mu, e$. The same is true for the form factor F_3. However, it does play a considerable role in the Ward identity.

3. Ward identities

The Ward Identities, which are derived in appendix, relate various form factors [30]

$$\begin{aligned}
\langle V(k,\varepsilon)|\bar{u}i\sigma^{\mu\nu}q_\nu b|B(p)\rangle &= -(m_b+m_q)\langle V(k,\varepsilon)|\bar{u}\gamma^\mu b|B(p)\rangle &(3.1)\\
\left\langle V(k,\varepsilon)\left|\bar{u}i\sigma^{\mu\nu}\gamma^5 q_\nu b\right|B(p)\right\rangle &= (m_b-m_q)\left\langle V(k,\varepsilon)\left|\bar{u}\gamma^\mu\gamma^5 b\right|B(p)\right\rangle +\\
&\quad+(p+k)^\mu\left\langle V(k,\varepsilon)\left|\bar{u}\gamma^5 b\right|B(p)\right\rangle &(3.2)\\
-iq_\mu\left\langle V(k,\varepsilon)\left|\bar{u}\gamma^\mu\gamma^5 b\right|B(p)\right\rangle &= i(m_b+m_q)\left\langle V(k,\varepsilon)\left|\bar{u}\gamma^5 b\right|B(p)\right\rangle . &(3.3)
\end{aligned}$$

Using the Ward Identity (3.1) in eqs. (2.1) and (2.4), and comparing the coefficient, we obtain

$$F_1\left(q^2\right)=\frac{m_b+m_q}{M_B+M_V}2V\left(q^2\right) \tag{3.4}$$

Again, using the Ward Identity (3.2) in eqs. (2.2) and (2.5), and comparing the coefficients of $\epsilon^{\mu *}$ and that of q^μ (we do not compare the coefficients of $(p+k)^\mu$ as heavy quark approximation has been used in obtaining the second term of eq. (6b)) we obtain

$$\begin{aligned}
F_2\left(q^2\right) &= \frac{m_b-m_q}{M_B-M_V}A_1\left(q^2\right) &(3.5)\\
F_3\left(q^2\right) &= -(m_b-m_q)\frac{2M_V}{q^2}\left[A_3\left(q^2\right)-A_0\left(q^2\right)\right] . &(3.6)
\end{aligned}$$

The results given in eqs. (3.5), (3.6) and (3.4) are model independent because these are derived by using Ward Identities.

In order to obtain a universal normalization of the above form factors at $q^2=0$, we define

$$\begin{aligned}
\langle V(k,\epsilon)|i\bar{u}\sigma_{\alpha\beta}b|B(p)\rangle &= -i\varepsilon_{\alpha\beta\rho\sigma}\varepsilon^{*\rho}\left[(p+k)^\sigma g_+ + q^\sigma g_-\right] - q\cdot\varepsilon^*(k)\,\varepsilon_{\alpha\beta\rho\sigma}(p+k)^\rho q^\sigma h -\\
&\quad -i\left[q_\alpha\varepsilon_{\beta\rho\sigma\tau}\varepsilon^{*\rho}(p+k)^\sigma q^\tau-\alpha\leftrightarrow\beta\right]h_1-\\
&\quad -i\left[(p+k)_\alpha\varepsilon_{\beta\rho\sigma\tau}\varepsilon^{*\rho}(p+k)^\sigma q^\tau-\alpha\leftrightarrow\beta\right]h_2 . &(3.7)
\end{aligned}$$

Since

$$\sigma^{\mu\nu}\gamma_5=-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} , \tag{3.8}$$

we can write

$$\langle V(k,\epsilon)|i\bar{u}\sigma^{\mu\nu}\gamma_5 b|B(p)\rangle=-\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\left\langle V\left(p'\right)|i\bar{u}\sigma_{\alpha\beta}b|B(p)\right\rangle \tag{3.9}$$

Using

$$\sigma^{\mu\nu}=\frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right],\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3,\ \epsilon^{0123}=-1$$

and

$$\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\delta\rho\sigma}=-\begin{vmatrix} g_\delta^\alpha & g_\rho^\alpha & g_\sigma^\alpha\\ g_\delta^\beta & g_\rho^\beta & g_\sigma^\beta\\ g_\delta^\nu & g_\rho^\nu & g_\sigma^\nu\end{vmatrix}$$

we obtain from eq. (3.7)

$$\begin{aligned}\langle V(k,\epsilon)| i\bar{u}\sigma^{\mu\nu}q_\nu\gamma_5 b| B(p)\rangle =& \varepsilon^{*\mu}\left[\left(M_B^2-M_V^2\right)g_+ + q^2 g_-\right]- \\ & -q\cdot\varepsilon^*(k)\left[(p+k)^\mu g_+ + q^\mu g_-\right]+ \\ & +q\cdot\varepsilon^*(k)\left[q^2(p+k)^\mu-\left(M_B^2-M_V^2\right)q^\mu\right]h- \\ & -\varepsilon^{*\mu}\left(M_B^2-M_V^2\right)^2 h_2+\varepsilon^{*\mu}\left(2M_B^2+2M_V^2-q^2\right)q^2 h_2+ \\ & +q\cdot\varepsilon^*(k)(p+k)^\mu\left(M_B^2-M_V^2-q^2\right)h_2+ \\ & +q\cdot\varepsilon^*(k)q^\mu\left(-M_B^2-3M_V^2+q^2\right)h_2 \end{aligned} \quad (3.10)$$

Equating eqs. (2.5) and (3.10) and comparing various coefficients

$$F_3\left(q^2\right) = -g_- - \left(M_B^2-M_V^2\right)h - \left(M_B^2+3M_V^2-q^2\right)h_2 \quad (3.11)$$

$$\begin{aligned}F_2\left(q^2\right) &= g_+ - q^2 h - \left(M_B^2-M_V^2-q^2\right)h_2 - \frac{q^2}{M_B^2-M_V^2}F_3\left(q^2\right) \quad (3.12)\\ &= g_+ + \frac{q^2}{M_B^2-M_V^2}g_- - \left[M_B^2-M_V^2-\frac{q^2}{M_B^2-M_V^2}\left(2M_B^2+2M_V^2-q^2\right)\right]h_2\,.\end{aligned}$$

Further, from eq. (3.7)

$$\left\langle V(k,\epsilon)\left| i\bar{u}\sigma_{\alpha\beta}q^\beta b\right| B(p)\right\rangle = -i\varepsilon_{\alpha\beta\rho\sigma}\varepsilon^{*\rho}p^\sigma q^\beta 2\left[g_+ - q^2 h_1 - \left(M_B^2-M_V^2\right)h_2\right]. \quad (3.13)$$

Comparing eqs. (3.13) and (2.4), we obtain

$$F_1\left(q^2\right) = 2\left[g_+ - q^2 h_1 - \left(M_B^2-M_V^2\right)h_2\right]. \quad (3.14)$$

Note that above relations ensure that $F_2(0) = (1/2)F_1(0)$. Equation (3.5), by using the value of $F_2\left(q^2\right)$, becomes

$$\begin{aligned}A_1(q^2) =& \frac{M_B-M_V}{m_b-m_q}\times \quad (3.15)\\ & \times\left\{g_+ + \frac{q^2}{M_B^2-M_V^2}g_- - \left[M_B^2-M_V^2-\frac{q^2}{M_B^2-M_V^2}\left(2M_B^2+2M_V^2-q^2\right)\right]h_2\right\}.\end{aligned}$$

Equation (3.14), by using the value of $F_1\left(q^2\right)$ given in eq. (3.4), becomes

$$V\left(q^2\right) = \frac{M_B+M_V}{m_b+m_q}\left[g_+ - q^2 h_1 - \left(M_B^2-M_V^2\right)h_2\right]. \quad (3.16)$$

Rearranging eq. (2.3), we get

$$\begin{aligned}A_2(q^2) &= \frac{M_B+M_V}{M_B-M_V}A_1\left(q^2\right) - \frac{2M_V}{M_B-M_V}A_3\left(q^2\right) \\ &= \frac{M_B+M_V}{M_B-M_V}A_1\left(q^2\right) - \frac{2M_V}{M_B-M_V}\left[A_3\left(q^2\right)-A_0\left(q^2\right)\right] - \frac{2M_V}{M_B-M_V}A_0\left(q^2\right) \\ &= \frac{M_B+M_V}{m_b-m_q}\left\{g_+ - q^2 h - \left[M_B^2-M_V^2+\frac{q^2}{M_B^2-M_V^2}\left(3M_B^2+5M_V^2-2q^2\right)\right]h_2\right\} - \\ &\quad -\frac{2M_V}{M_B-M_V}A_0\left(q^2\right) \end{aligned} \quad (3.17)$$

where we have used eqs. (3.6) and (3.11).

The merit of above relations is that the normalization of the form factors A_1, V, F_1 and F_2 at $q^2 = 0$ is determined by a single constant $\left[g_+ (0) - \left(M_B^2 - M_V^2\right) h_2 (0)\right]$ while those of A_2 and A_0 are determined by $\left[g_+ (0) - \left(M_B^2 - M_V^2\right) h_2 (0)\right]$ and $A_0 (0)$.

4. Pole contribution

Only h_1, g_-, h, and A_0 get pole contributions from $B^* (1^-)$, $B_A^* (1^+)$, and $B (0^-)$ mesons. These are given by

$$h_1|_{pole} = -\frac{1}{2}\frac{g_{B^* B\rho^-}}{M_{B^*}^2}\frac{f_T^{B^*}}{1 - q^2/M_{B^*}^2} = \frac{R_V}{M_{B^*}^2}\frac{1}{1 - q^2/M_{B^*}^2} \tag{4.1}$$

$$g_-|_{pole} = -\frac{g_{B_A^* B\rho^-}}{M_{B_A^*}^2}\frac{f_T^{B_A^*}}{1 - q^2/M_{B_A^*}^2} = \frac{R_A^S}{M_{B_A^*}^2}\frac{1}{1 - q^2/M_{B_A^*}^2} \tag{4.2}$$

$$h|_{pole} = \frac{1}{2}\frac{f_{B_A B\rho^-}}{M_{B_A^*}^2}\frac{f_T^{B_A^*}}{1 - q^2/M_{B_A^*}^2} = \frac{R_A^D}{M_{B_A^*}^2}\frac{1}{1 - q^2/M_{B_A^*}^2} \tag{4.3}$$

$$A_0 \left(q^2\right)|_{pole} = \frac{g_{BB\rho^-}}{M_\rho} f_B \frac{q^2/M_B^2}{1 - q^2/M_B^2} = R_0 \frac{q^2/M_B^2}{1 - q^2/M_B^2}\,. \tag{4.4}$$

On the other hand, only g_+, g_-, and A_0 get contribution from quark $\triangle$–graph. Therefore we shall put $h_2 = 0$ in what follows.

The above coupling constants are defined as follows:

$$\begin{aligned}
\left\langle B^{*+} (\eta, q)\, \rho^- (\epsilon, k) \left| B^0 (p) \right.\right\rangle &= i g_{B^* B\rho^-} \epsilon_{\alpha\rho\mu\sigma} \epsilon^{*\alpha} q^\rho p^\sigma \eta^{*\mu} \\
\langle 0 \left| i\bar{u}\sigma_{\mu\nu} b \right| B^* (\eta, q)\rangle &= f_T^{B^*} (q_\mu \eta_\nu - q_\nu \eta_\mu) \\
\left\langle B_A^{*+} (\eta, q)\, \rho^- (\epsilon, k) \left| B^0 (p) \right.\right\rangle &= i g_{B_A^* B\rho^-} \epsilon^* \cdot \eta^* - f_{B_A^* B\rho^-} q \cdot \epsilon^* k \cdot \eta^* \\
\langle 0 \left| i\bar{u}\sigma_{\mu\nu} b \right| B^* (\eta, q)\rangle &= f_T^{B_A^*} \epsilon_{\mu\nu\alpha\beta} \eta^\alpha q^\beta\,.
\end{aligned} \tag{4.5}$$

Now if in the Ward Identity (3.1), we take the matrix elements between $\langle 0|$ and $|B^*\rangle$, we obtain

$$\langle 0 \left| i\bar{u}\sigma^{\mu\nu} q_\nu b \right| B^* (\eta, q)\rangle = - (m_b + m_q) f_{B^*} \eta^\mu \tag{4.6}$$

where $\langle 0 \left| i\bar{u}\gamma^\mu b \right| B^* (\eta, q)\rangle = f_{B^*} \eta^\mu$. Thus using eq. (4.5), we obtain

$$f_T^{B^*} = \frac{m_b + m_q}{M_{B^*}^2} f_{B^*} = \frac{m_b + m_q}{M_{B^*}} f_B = \frac{M_B}{M_{B^*}} f_B = f_B\,. \tag{4.7}$$

It is easy to see that same relation is obtained if one uses heavy quark spin symmetry. Similarly, if in the Ward Identity (3.2), we take the matrix elements between $\langle 0|$ and $|B_A^*\rangle$, we obtain on using that $\langle 0 \left| i\bar{u}\gamma_5 b \right| B_A^* (\eta, q)\rangle = 0$,

$$\langle 0 \left| i\bar{u}\sigma^{\mu\nu} q_\nu \gamma_5 b \right| B_A^* (\eta, q)\rangle = (m_b - m_q) f_{B_A^*} \eta^\mu\,, \tag{4.8}$$

where $\langle 0 \left| i\bar{u}\gamma^\mu \gamma_5 b \right| B_A^* (\eta, q)\rangle = f_{B_A^*} \eta^\mu$. By using the identity (3.8), we obtain

$$f_T^{B_A^*} = -\frac{m_b - m_q}{M_{B_A^*}^2} f_{B_A^*}\,. \tag{4.9}$$

5. Form factors and determination of coupling constants

The form factors $V\left(q^2\right)$ (eq. (3.16)), $A_1\left(q^2\right)$ (eq. (3.15)), and $A_2\left(q^2\right)$ (eq. (3.17)) can be written as

$$\begin{aligned}
V\left(q^2\right) &= \frac{M_B+M_V}{m_b+m_q}\left\{g_+\left(q^2\right)-q^2h_1\left(q^2\right)\right\} \\
&\simeq \frac{M_B+M_V}{M_B}\left\{g_+\left(q^2\right)-R_V\frac{q^2}{M_{B^*}^2}\frac{1}{1-q^2/M_{B^*}^2}\right\} \qquad (5.1)\\
A_1\left(q^2\right) &= \frac{M_B-M_V}{m_b-m_q}\left\{g_+\left(q^2\right)+\frac{q^2}{M_B^2-M_V^2}g_-\left(q^2\right)\right\} \\
&= \frac{M_B-M_V}{m_b-m_q}\left\{g_+\left(q^2\right)+\frac{q^2}{M_B^2-M_V^2}\left(\tilde{g}_-\left(q^2\right)+g_-\left(q^2\right)\big|_{pole}\right)\right\} \\
&\simeq g_+\left(q^2\right)+\frac{q^2}{M_B^2-M_V^2}\tilde{g}_-\left(q^2\right)+\frac{R_A^S}{M_B^2-M_V^2}\frac{q^2}{M_{B_A^*}^2}\frac{1}{1-q^2/M_{B_A^*}^2} \qquad (5.2)\\
A_2\left(q^2\right) &= \frac{M_B+M_V}{m_b-m_q}\left\{g_+\left(q^2\right)-q^2h\left(q^2\right)\right\}-\frac{2M_V}{M_B-M_V}A_0\left(q^2\right) \\
&\simeq \frac{M_B+M_V}{M_B-M_V}\left\{g_+\left(q^2\right)-R_A^D\frac{q^2}{M_{B_A^*}^2}\frac{1}{1-q^2/M_{B_A^*}^2}\right\}-\frac{2M_V}{M_B-M_V}A_0\left(q^2\right). \qquad (5.3)
\end{aligned}$$

The behaviour of $g_+\left(q^2\right)$, $\tilde{g}_-\left(q^2\right)$ and $A_0\left(q^2\right)$ near $q^2\to 0$ is known from LEET

$$g_+\left(q^2\right) = \frac{\xi_\perp(0)}{\left(1-q^2/M_B^2\right)^2} = -\tilde{g}_-\left(q^2\right) \qquad (5.4)$$

$$A_0\left(q^2\right) = \left(1-\frac{M_V^2}{M_BE_V}\right)\xi_\parallel\left(q^2\right)+\frac{M_V}{M_B}\xi_\perp\left(q^2\right) \qquad (5.5)$$

$$A_0(0) = \frac{M_B^2-M_V^2}{M_B^2+M_V^2}\xi_\parallel(0)+\frac{M_V}{M_B}\xi_\perp(0) \qquad (5.6)$$

$$E_V = \frac{M_B}{2}\left(1-\frac{q^2}{M_B^2}+\frac{M_V^2}{M_B^2}\right). \qquad (5.7)$$

The pole terms in the relations (5.1), (5.2) and (5.3) are expected to dominate near $q^2 = M_{B^*}^2$ or $M_{B_A^*}^2$. Therefore, we cannot expect these relations, obtained from Ward identities, to hold for all q^2 for which we use the parametarization, suggested by above behavior, near $q^2=0$ and near the pole

$$F\left(q^2\right) = \frac{F(0)}{\left(1-q^2/M^2\right)\left(1-q^2/M'^2\right)}, \qquad (5.8)$$

where M^2 is $M_{B^*}^2$ or $M_{B_A^*}^2$ and M' is the radial excitation of M. This parameterization takes into account potential corrections to single pole dominance, presumably arising from radial excitations of M [30], as suggested by dispersion relations [31, 32] (a detailed discussion on this point one can find in [30]). It also takes care of off-mass-shell-ness of couplings of B^* or B_A^* with $B\rho$ channel [30] by replacing the pole contribution

$$\frac{R}{1-q^2/M^2}\ \textit{with}\ \frac{R\left(1-M^2/M'^2\right)}{\left(1-q^2/M^2\right)\left(1-q^2/M'^2\right)}. \qquad (5.9)$$

	J^P	M	M'/M
M_B	0^-	5.28	1.14
M_{B^*}	1^-	5.33	1.14
$M_{B_A^*}$	1^+	5.71	1.12

Table 1: B-mesons masses in GeV [12, 24].

Since $g_+\left(q^2\right)$ and $\tilde{g}_-\left(q^2\right)$ do not have a pole at $q^2 = M_{B^*}^2$, we get

$$V\left(q^2\right)\left(1 - q^2/M_{B^*}^2\right)\Big|_{q^2=M_{B^*}^2} = -\frac{M_B + M_V}{M_B} R_V\,.$$

This becomes

$$R_V \equiv -\frac{1}{2} g_{B^* B\rho^-} f_T^{B^*} = -\frac{1}{2} g_{B^* B\rho^-} f_B = -\frac{g_+(0)}{1 - M_{B^*}^2/M_{B^*}'^2}\,. \tag{5.10}$$

Similarly, using parameterization (5.8) for the first term on the right hand side of eq. (5.3),

$$R_A^D \equiv -\frac{1}{2} f_{B_A^* B\rho^-} f_T^{B_A^*} = \frac{1}{2} f_{B_A^* B\rho^-} \frac{M_B - M_V}{M_{B_A^*}^2} f_{B_A^*} = -\frac{g_+(0)}{1 - M_{B_A^*}^2/M_{B_A^*}'^2}\,. \tag{5.11}$$

We cannot use the parameterization (5.8) for $A_1\left(q^2\right)$ since near $q^2 = 0$, $A_1\left(q^2\right)$ behaves as $g_+\left(q^2\right)\left[1 - q^2/\left(M_B^2 - M_V^2\right)\right]$. This suggests the following

$$A_1\left(q^2\right) = \frac{g_+(0)}{\left(1 - q^2/M_{B_A^*}^2\right)\left(1 - q^2/M_{B_A^*}'^2\right)}\left(1 - \frac{q^2}{M_B^2 - M_V^2}\right) \tag{5.12}$$

so that, since g_+ and $\tilde{g}_-$ do not have pole at $q^2 = M_{B_A^*}^2$,

$$\begin{aligned} A_1\left(q^2\right)\left[1 - \frac{q^2}{M_{B_A^*}^2}\right]\Bigg|_{q^2=M_{B_A^*}^2} &= g_+(0)\,\frac{1 - M_{B_A^*}^2/\left(M_B^2 - M_V^2\right)}{1 - M_{B_A^*}^2/M_{B_A^*}'^2} \\ &= \frac{R_A^s}{M_B^2 - M_V^2} M_{B_A^*}^2 = -g_{B_A^* B\rho^-} \frac{M_B - M_V}{M_{B_A^*}^2} f_{B_A^*} \frac{1}{M_B^2 - M_V^2}\,. \end{aligned} \tag{5.13}$$

Most of the estimates of $g_+(0)$ suggest that [1]

$$g_+(0) = 0.29 \pm 0.04\,[11] \tag{5.14}$$

For numerical work we shall use the B-meson masses given in table 1. Thus we have the prediction from eq. (5.10)

$$g_{B^* B\rho^-} = \frac{2\,(1.26 \pm 0.17)}{f_B}\,. \tag{5.15}$$

[1] For example, the LCSR calculations gave $g_+(0) = 0.29 \pm 0.04$ [11], 0.24 ± 0.07 [33, 9], 0.30 ± 0.10 [34], 0.28 [35], and 0.27 [36].

This prediction ($f_B = 180\,\mathrm{MeV}$) is consistant with its various estimates e.g. QCD sum rules give it $\sqrt{2}\,(11)\,\mathrm{GeV}^{-1}$ with $g_{B^*B\rho} = \sqrt{2} g_{B^*B\rho^0}$ (where $g_{B^*B\rho^0}$ has been estimated in [32]) and the constituent quark model (CQM) predicts it to be [31]

$$\sqrt{2}\left\{\frac{1}{2}\left(\frac{1}{m_b}+\frac{1}{m_u}\right)\frac{M_\rho^2}{f_{\rho^-}}\sqrt{2}\right\} \simeq \sqrt{2}\,(9.75)\ \mathrm{GeV}^{-1}\,. \tag{5.16}$$

The predictions (5.11) and (5.13) cannot be tested at present since we do not have any information on $f_{B_A^*}$. However, the ratio of S-wave and D-wave couplings is predicted to be

$$\begin{aligned}\frac{g_{B_A^*B\rho^-}}{f_{B_A^*B\rho^-}} &= \frac{M_{B_A^*}^2 - \left(M_B^2 - M_V^2\right)}{2} \\ g_{B_A^*B\rho^-} &= 2.66 \times f_{B_A^*B\rho^-}\ \mathrm{GeV}^2\,.\end{aligned} \tag{5.17}$$

The different values of $F\,(0)$'s are

$$V\,(0) = \frac{M_B + M_V}{M_B} g_+\,(0) \tag{5.18}$$

$$A_1\,(0) = g_+\,(0) \tag{5.19}$$

$$A_2\,(0) = \frac{M_B + M_V}{M_B - M_V} g_+\,(0) - \frac{2M_V}{M_B - M_V} A_0\,(0) \tag{5.20}$$

where $g_+\,(0)$ is given by (5.14). As far as $A_0\,(0)$ is concerned we can get some infor mation by writing

$$\begin{aligned}A_0\left(q^2\right) &= \tilde{A}_0\left(q^2\right) + A_0\left(q^2\right)\big|_{pole} \\ &= \tilde{A}_0\left(q^2\right) + R_0 \frac{q^2/M_B^2\left(1 - M_B^2/M_B'^2\right)}{\left(1 - q^2/M_B^2\right)\left(1 - q^2/M_B'^2\right)}\,.\end{aligned} \tag{5.21}$$

Using the parameterization for $A_0\left(q^2\right)$ similar to (5.8) and the same argument as before (cf. eq. (5.10)), we get

$$\begin{aligned}A_0\,(0) &= \tilde{A}_0\,(0) = R_0\left(1 - \frac{M_B^2}{M_B'^2}\right) \\ &= \frac{g_{BB\rho^-}}{M_\rho} f_B\left(1 - \frac{M_B^2}{M_B'^2}\right).\end{aligned} \tag{5.22}$$

Using the ρ-dominance

$$g_{BB\rho^-} = \frac{M_\rho^2}{f_{\rho^-}}\frac{1}{F_\rho\,(0)} \tag{5.23}$$

so that

$$A_0\,(0) = f_B \frac{M_\rho}{f_{\rho^-}}\left(1 - \frac{M_B^2}{M_B'^2}\right)\frac{1}{F_\rho\,(0)} \tag{5.24}$$

where $F_\rho\,(0)$ takes care off mass shell-ness of ρ-meson and is given by [31]

$$F_\rho\,(0) = \left(\frac{g_{\rho\pi\pi} f_{\rho^-}}{\sqrt{2} M_\rho^2}\right)_{exp}^{-1} = 0.80\,, \tag{5.25}$$

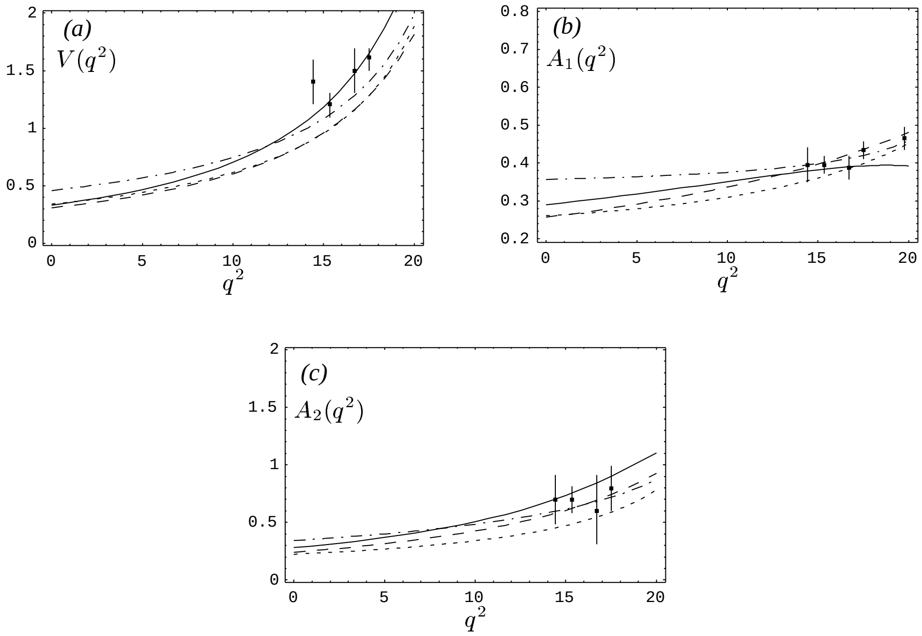

Figure 1: Comparison of the form factor vs. lattice data [41, 42] and calculations within different approaches is given. Solid line-our result, dotted line-LCSR [11], dashed line-quark model [14], dash-dotted line-lattice constrained parametriation of [7].

where we have used

$$\left(\frac{g_{\rho\pi\pi} f_{\rho^-}}{\sqrt{2} M_\rho^2}\right)_{exp} = 1.22 \pm 0.03. \tag{5.26}$$

Using $f_B = 180\,\mathrm{MeV}$, $f_{\rho^-}/M_\rho = 198\,\mathrm{MeV}$,

$$A_0\left(0\right) = 0.26\,. \tag{5.27}$$

Although LEET does not give any relationship between $\xi_{||}\left(0\right)$ and $\xi_\perp\left(0\right)$ but due to some numerical coincidence in the LCSR expressions for $\xi_{||}$ and $\xi_\perp$ [20][2]

$$\xi_{||}\left(0\right) \simeq \xi_\perp\left(0\right) = g_+\left(0\right) \tag{5.28}$$

so that, from eq. (5.6)

$$A_0\left(0\right) = 1.10 g_+\left(0\right) \simeq 0.320 \pm 0.044 \tag{5.29}$$

consistent with the above estimate (5.27). Another estimate of $A_0\left(0\right)$ in light-cone sum rules (LCSR) approach is 0.372.

We summarise the form factors which we shall use for our numerical work ($M_V = M_\rho = 0.770\,\mathrm{GeV}$)

$$V\left(q^2\right) = \frac{V\left(0\right)}{\left(1 - q^2/M_B^2\right)\left(1 - q^2/M_B'^2\right)} \tag{5.30}$$

[2]With $\xi_{||} = \xi_\perp$, Soares [37] finds a good agreement between his prediction and the lattice data [38]. In the Bakamjian-Thomas formalism [39], $\xi_{||} = \xi_\perp$ would hold if the D-wave were asymptotically suppressed by some power of E with respect to the S-wave in the vector meson [40].

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$$A_1\left(q^2\right) = \frac{A_1\left(0\right)}{\left(1-q^2/M_{B_A^*}^2\right)\left(1-q^2/M_{B_A^*}^{\prime 2}\right)}\left(1-\frac{q^2}{M_B^2-M_V^2}\right) \tag{5.31}$$

$$A_2\left(q^2\right) = \frac{\tilde{A}_2\left(0\right)}{\left(1-q^2/M_{B_A^*}^2\right)\left(1-q^2/M_{B_A^*}^{\prime 2}\right)} - \frac{2M_V}{M_B-M_V}\frac{A\left(0\right)}{\left(1-q^2/M_B^2\right)\left(1-q^2/M_B^{\prime 2}\right)} \tag{5.32}$$

where

$$\begin{aligned} V\left(0\right) &= 0.332 \pm 0.046\,, \\ A_1\left(0\right) &= 0.29 \pm 0.04\,, \\ \tilde{A}_2\left(0\right) &= 0.389 \pm 0.054\,. \end{aligned} \tag{5.33}$$

These form factors (5) are plotted in figure 1, where the comparison of the form factor vs. lattice data [7, 41, 42] and calculations within different approaches is given. Solid line-our result, dotted line-LCSR [11], dashed line-quark model [14], dash-dotted line-lattice constrained parametriation of [7].

6. Decay distribution

Four independent kinematical variables completely describe the semileptonic decay $B \to \rho l \bar{\nu}$, where the vector meson decays to two pseudoscalar mesons, $\rho \to \pi\pi$. The four variables commonly used are $q^2 = (p-k)^2$ and the three angles (θ_l, θ_ρ, and χ). The angle θ_l is measured in the W^* (or $l\bar{\nu}$) rest frame, it is the polar angle between the charged lepton and the direction opposite to that of vector meson. The angle θ_ρ is measured in the rest frame of vector meson, where the pseudoscalar mesons are back to back, and it is the polar angle between one of these mesons. The angle χ is chosen to be azimuthal angle between the plane of flight of lepton and of pseudoscalar mesons.

The differential decay rate for $B \to \rho l \nu$, $\rho \to \pi\pi$ can be expressed in terms of these four variables q^2, θ_l, θ_ρ, and χ [43, 44]

$$\begin{aligned} \frac{d\Gamma\left(B\to\rho l\nu,\rho\to\pi\pi\right)}{dq^2\,d\cos\theta_\rho\,d\cos\theta_l\,d\chi} = {} & \frac{3}{8\left(4\pi\right)^4}G_F^2\left|V_{ub}\right|^2\frac{\left|\mathbf{k}_\rho\right|q^2}{M_B^2}\mathcal{B}\left(\rho\to\pi\pi\right) \\ & \times\Big\{(1-\cos\theta_l)^2\sin^2\theta_\rho|H_+(q^2)|^2+(1+\cos\theta_l)^2\sin^2\theta_\rho|H_-(q^2)|^2+ \\ & \quad + 4\sin^2\theta_l\cos^2\theta_\rho|H_0(q^2)|^2 - \\ & \quad - 4\sin\theta_l(1-\cos\theta_l)\sin\theta_\rho\cos\theta_\rho\cos\chi H_+(q^2)H_0(q^2) + \\ & \quad + 4\sin\theta_l(1+\cos\theta_l)\sin\theta_\rho\cos\theta_\rho\cos\chi H_-(q^2)H_0(q^2) - \\ & \quad - 2\sin^2\theta_l\sin^2\theta_\rho\cos 2\chi H_+(q^2)H_-(q^2)\Big\} \end{aligned} \tag{6.1}$$

where

$$\left|\mathbf{k}_\rho\right| = \sqrt{\frac{\left(M_B^2+M_\rho^2-q^2\right)^2}{4M_B^2}-M_\rho^2} \tag{6.2}$$

is the magnitude of three-momentum of ρ in the rest frame of B and is a function of q^2.

Bescause the parent meson has spin zero, the vector meson and the W^* must have the same helicity. The amplitudes for the helicities 0, +1, and -1 are proportional to $H_0\left(q^2\right)$, $H_+\left(q^2\right)$, and $H_-\left(q^2\right)$. The detailed dynamics of the hadronic current are described by the variation of these helicity amplitudes with q^2. Equation (6.1) incorporates the $V-A$ structure of the leptonic current, as well as the assumption that the mass of the charged lepton can be neglected.

The helicity amplitudes can in turn be related to the two axial form factors, $A_1\left(q^2\right)$ and $A_2\left(q^2\right)$, and the vector form factor $V\left(q^2\right)$, which appear in the hadronic current (2.2) and (2.1) respectively,

$$H_0\left(q^2\right)=\frac{1}{2M_\rho\sqrt{q^2}}\left[\left(M_B^2-q^2-M_\rho^2\right)\left(M_B+M_\rho\right)A_1\left(q^2\right)-\frac{4M_B^2\left|\mathbf{k}_\rho\right|^2}{M_B+M_\rho}A_2\left(q^2\right)\right] \tag{6.3}$$

and

$$H_\pm\left(q^2\right)=\left(M_B+M_\rho\right)A_1\left(q^2\right)\mp\frac{2M_B\left|\mathbf{k}_\rho\right|}{M_B+M_\rho}V\left(q^2\right). \tag{6.4}$$

The form factors $A_1\left(q^2\right)$, $A_2\left(q^2\right)$, and $V\left(q^2\right)$ are dimensionless and relatively real, since CP is conserved in these decays and there are no final state strong interactions [16]. We note that, while A_2 contributes only to H_0 and V contributes only to $H_\pm$, A_1 contributes to all three helicity amplitudes. At high q^2 (small $|\mathbf{k}_\rho|$), each of the helicity amplitudes is dominated by A_1. At $q^2=0$, the over all factor of q^2 in (6.1) causes all the contributions to the decay rate to vanish except that from $\left|H_0\left(q^2\right)\right|^2$, since H_0 contains a factor of $1/\sqrt{q^2}$ and at $q^2=q^2_{\max}=\left(M_B-M_\rho\right)^2$, the over all factor of $|\mathbf{k}_\rho|$ vanishes, causing the rate to do likewise.

Integrating (6.1) over the angles θ_ρ and χ, we obtain

$$\begin{aligned}\frac{d^2\Gamma}{dq^2dE_l} &= \frac{G_F^2|V_{ub}|^2}{128\pi^3}\frac{q^2}{M_B^2}\{(1+\cos\theta_l)^2|H_-(q^2)|^2+(1-\cos\theta_l)^2|H_+(q^2)|^2+ \\ &\qquad +2(1-\cos^2\theta_l)|H_0(q^2)|^2\Big\}\end{aligned} \tag{6.5}$$

where θ_l is the polar angle between the ρ and the lepton l in the $(l\bar{\nu}_l)$ center of mass system, and

$$2M_B\left|\mathbf{k}_\rho\right|\cos\theta_l=M_B^2-M_\rho^2+q^2-4M_BE_l\,. \tag{6.6}$$

The E_l (or $\cos\theta_l$) integration of eq. (6.5) can be done trivially and results in the differential q^2-distribution

$$\frac{d\Gamma}{dq^2}=\frac{G_F^2\left|V_{ub}\right|^2}{96\pi^3}\left|\mathbf{k}_\rho\right|\frac{q^2}{M_B^2}\mathcal{B}\left(\rho\to\pi\pi\right)\left\{\left|H_-\left(q^2\right)\right|^2+\left|H_+\left(q^2\right)\right|^2+\left|H_0\left(q^2\right)\right|^2\right\}. \tag{6.7}$$

By numerically integrating eq. (6.5) over lepton energy E_l in the interval $0\le E_l\le 2.6$ and over q^2 in the interval

$$0\le q^2\le 2E_l\left(\frac{M_B^2-M_\rho^2-2M_BE_l}{M_B-2E_l}\right),$$

the width of $B \to \rho l \nu$ turns out to be

$$\Gamma\left(B \to \rho l \nu\right) = 15.82 \pm 2.18 \left|V_{ub}\right|^2 ps^{-1}\,, \tag{6.8}$$

where we have used the form factors given in eqs. (5).

The branching ratio of $B^0 \to \rho^- l^+ \nu$ is measured by CLEO collaboration [27], $\mathcal{B}_{expt}(B^0 \to \rho^- l^+ \nu) = \left(2.57 \pm 0.29^{+0.33}_{-0.46} \pm 0.41\right) \times 10^{-4}$, where the errors are statistical, systematic, and theoretical. The CKM matrix element $|V_{ub}|$ can be obtained from the branching ratio $\mathcal{B}_{expt}\left(B^0 \to \rho^- l^+ \nu\right)$ using

$$\begin{aligned} |V_{ub}| &= \sqrt{\frac{\mathcal{B}_{expt}\left(B^0 \to \rho^- l^+ \nu\right)}{\Gamma_{thy}\left(B^0 \to \rho^- l^+ \nu\right)\ \tau_{B^0}}} \\ &= 3.24 \times 10^{-3}\,. \end{aligned} \tag{6.9}$$

We use $\tau_{B^0} = 1.548 \pm 0.032$ ps [45]. Recently, the BaBar Collaboration [28] predicted the branching ratio $\mathcal{B}_{expt}(B^0 \to \rho^- l^+ \nu) = (3.29 \pm 0.42 \pm 0.47 \pm 0.60) \times 10^{-4}$ and $|V_{ub}| = \left(3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}\right) \times 10^{-3}$, where the uncertainities are statistical, systematic, and theoretical respectively. By using the branching ratio predicted by BaBar, and decay width (6.8), we obtain

$$|V_{ub}| = 3.67 \times 10^{-3}\,. \tag{6.10}$$

Another recent study by CLEO [29], gave the branching ratio $\mathcal{B}\left(B^0 \to \rho^- l^+ \nu\right) = (2.17 \pm 0.34^{+0.47}_{-0.54} \pm 0.41 \pm 0.01) \times 10^{-4}$, $|V_{ub}| = \left(3.00 \pm 0.21^{+0.29}_{-0.35}{}^{+0.49}_{-0.38} \pm 0.28\right) \times 10^{-3}$, where the errors in CLEO analyses are statistical, experimental systematic, theoretical systematicbased on lattice QCD and LCSR uncertainities, and $\rho l \nu$ form factor shape uncertainity, respectively. By using this branching ratio given by CLEO [29], we obtain

$$|V_{ub}| = 2.98 \times 10^{-3}\,.$$

In this study of $B \to \rho l \nu$ decay, there is only one free parameter i.e. $g_+(0)$, and we have used the value of this predicted by a LCSR calculation [11] given in eq. (5.14). The error appear in our calculation from this parameter. The error shown in our result of $\Gamma\left(B \to \rho l \nu\right)$ in (6.8) is only 13.8%.

7. Numerical analyses and discussions

The values of the helicity amplitudes and form factors at $q^2 = 0$ and $q^2 = q^2_{\max}$ are given in tables 2, 3, and 4. The variation of form factors at $q^2 = 0$ and $q^2 = q^2_{\max}$ is shown in

	$H_+(0)$	$H_-(0)$	$H_-(0)/H_+(0)$	$\sqrt{q^2}H_0(0)$
Our	0.256	3.253	12.714	8.737
QM [14]	0.163	2.942	18.014	8.202
LCSR [11]	0.055	3.103	56.767	10.158
Lattice [7]	0.0898	4.224	47.020	10.914

Table 2: A comparison between different calculations is given between the helicity amplitudes H_-, H_+ (GeV) and their ratio H_-/H_+ at $q^2 = 0$.

	$H_+\left(q^2_{\max}\right)$	$H_-\left(q^2_{\max}\right)$	$H_0\left(q^2_{\max}\right)$	$\sqrt{q^2}H_0\left(q^2_{\max}\right)$
Our	2.362	2.362	2.362	10.652
QM [14]	2.952	2.952	2.952	13.313
LCSR [11]	2.782	2.782	2.782	12.549
Lattice [7]	2.826	2.826	2.826	12.744

Table 3: A comparison between different calculations is given between the helicity amplitudes H_-, H_+ (GeV) at $q^2 = q^2_{\max}$.

	V	A_1	A_2	$\frac{V\left(q^2_{\max}\right)}{V(0)}$	$\frac{A_1\left(q^2_{\max}\right)}{A_1(0)}$	$\frac{A_2\left(q^2_{\max}\right)}{A_2(0)}$
Our	0.332 (2.605)	0.29 (0.390)	0.279 (1.136)	7.840	1.346	4.06
QM [14]	0.308 (1.915)	0.257 (0.488)	0.242 (0.956)	6.218	1.901	3.955
LCSR [11]	0.338 (2.011)	0.261 (0.460)	0.223 (0.818)	5.948	3.135	2.062
Lattice [7]	0.458 (2.102)	0.356 (0.467)	0.342 (0.898)	4.587	1.310	2.628

Table 4: Variation of form factors at $q^2 = 0$ $\left(q^2 = q^2_{\max}\right)$ is shown. A comparison of the ratio of respective form factors at $q^2 = q^2_{\max}$ over at $q^2 = 0$ is presented.

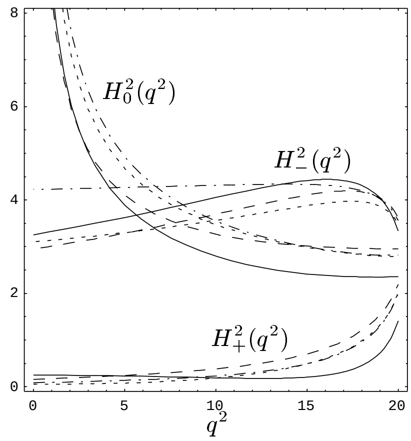

Figure 2: Comparison of Helicity amplitudes vs calculations within different approaches is given. Line description is as in figure 1.

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table 4. None of the helicity amplitudes changes sign over the entire physical range of q^2 as shown in figure 2. Although H_+ and H_0 both get contributions from two form factors which enters with opposite signs. The sqaure of the helicity amplitudes is plotted in figure 3.

First, H_- is generally dominent over H_+, while the condition $H_- = H_+$ is forced by kinematics at $q^2 = q^2_{\max}$; these two amplitudes go their own way and differ by more than a factor of 12.7 at $q^2 = 0$, a comparison over this point is given in table 2.Again kinematics forces $H_0 = H_- = H_+$ at $q^2 = q^2_{\max}$, but the amplitudes soon separate so that in the region near $q^2 = 0$ the amplitude H_0, which contain a factor of $1/\sqrt{q^2}$, very much dominates the others. This feature is independent of modest changes in the form factors.

We have used vector-current form factor pole at 5.33 GeV and axial-vector current form factor pole at 5.71 GeV. Note that the poles are very close to the edge of physical

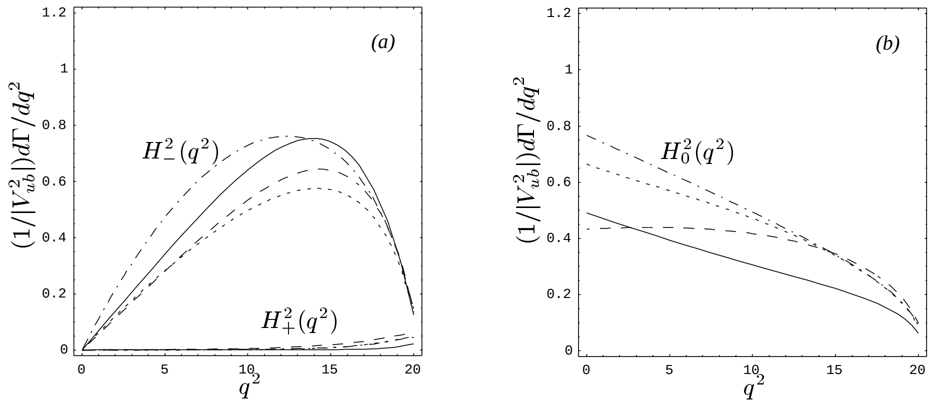

Figure 3: $d\Gamma/dq^2$ distribution for each of the three terms in eq. (6.7): (a) the terms proportional to $|H_-|^2$ and $|H_+|^2$, (b) the terms proportional to $|H_0|^2$. Line description is as in figure 1.

	Γ_-	Γ_+	$\Gamma_L = \Gamma_0$	Γ_T	Γ_{Total}	Γ_L/Γ_T
Our	9.662	0.059	6.095	9.721	15.82	0.63
QM [14]	-	-	-	-	15.8±2.3	0.88±0.08
LCSR [9]	-	-	-	-	13.5±4.0	0.52±0.08
Lattice [7]	-	-	-	-	$16.5^{+3.5}_{-2.3}$	$0.80^{+0.04}_{-0.03}$
ISGW2 [13]	-	-	-	-	14.2	0.3

Table 5: Different decay rate values are calculated and compared with different approaches. The comparison of Γ_L/Γ_T is presented and compare with other approaches.

region. This means that on the one hand, they should indeed dominate the behavior of the form factors near $q^2 = q^2_{\max}$. The change in the q^2 range in different form factors give quite distinct results as shown in table 4.

The H_+ (H_-) helicity amplitudes gets destructive (constructive) contributions from the A_1 and V form factors. The near zero behavior in H_+ arises, since V with its coefficient $2M_B\,|\mathbf{k}_\rho|\,/\,(M_B + M_\rho)$ in eq. (6.4) causes its contribution to this halicity amplitude to quickly catch up to that of A_1 as we move away from $q^2 = q^2_{\max}$. Of course this increases H_- correspondingly, according to eq. (6.4).

A similar situation pertains to the helicity amplitude H_0, where A_1 and A_2 destructively interfere, as shown in eq. (6.3). This pulls the whole longitudnal portion of the decay rate ($\Gamma_L = \Gamma_0$) down and it is far smaller than the transverse one ($\Gamma_T = \Gamma_+ + \Gamma_-$), see table 5. This agrees with the observations of Gilman and Singleton [43], and Körner and Schuler [16] (as shown in eq. (19) of ref. [16]) but contradicts the recent observation of Ali and Safir [46] in the LEET approach.

As an exercise, we check the relation among the form factors which holds true in large energy limit (LEL) (for a recent discussion see refs. [20, 47, 48, 49]), namely

$$\frac{A_1\left(q^2\right)}{V\left(q^2\right)} = \frac{2E_V M_B}{\left(M_B + M_V\right)^2} = \frac{M_B^2 + M_V^2 - q^2}{\left(M_B + M_V\right)^2}. \tag{7.1}$$

In figure 4, we plot the ratio of $B \to \rho$ form factors $A_1\left(q^2\right)/V\left(q^2\right)$ and compare them with

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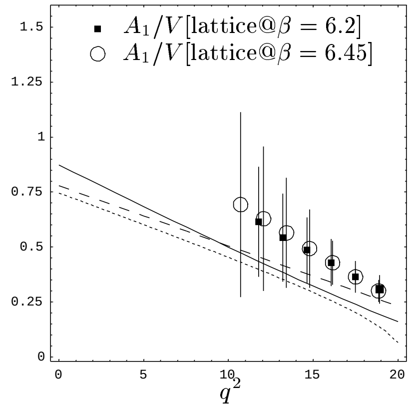

Figure 4: Ratio of $B \to \rho$ form factors, $A_1\left(q^2\right)/V\left(q^2\right)$, are plotted with lattice data points [50]. The solid line is our result. The dashed line is the LEET prediction [20] and the dotted line is the prediction of [37, 21].

Observable	V/A_1 $q^2=0$	V/A_1 $q^2 = 14.53\,\mathrm{GeV}^2$	V/A_1 $q^2 = 14.68\,\mathrm{GeV}^2$	Γ_+/Γ_-
Our	1.146	2.96	3.00	0.006
HQET-LEET [20]	1.29	2.63	2.66	0.02
Lattice [50]	-	2.05	2.02	-

Table 6: The values of the V/A_1 at different q^2 is given and compared with HQET-LEET approach and with recent Lattice data. Also ratio Γ_+/Γ_- is given.

the LEL [20] eq. (7.1) (dashed line), with the result of ref. [37, 21] (dotted line) and the lattice results [50], whereas the solid line is the result of this calculation. The knowledge of this ratio has important consequences: on the one hand, its measurment at $q^2 = 0$ will provide a test of various approaches; on the other hand it provides the ratio Γ_+/Γ_- of the width to helicity eigenstates $\lambda = \pm 1$. In table 6, the values of the V/A_1 at different q^2 is given and compared with HQET-LEET [20] approach and with recent Lattice data [50]. Also ratio Γ_+/Γ_- is given in table 6.

8. Conclusions

We have studied $B \to \rho l \nu_l$ decay by using Ward Identities. The form factors have been calculated and found that their normalization is essentially determined by a single constant $g_+(0)$. Then we use a parametrazation (eq. (5.8)) which take into account potential corrections to single pole dominance arising from radial excitations of M (where $M = M_{B^*}$ or $M_{B_A^*}$), as suggested by dispersion relations [31, 32]. This also takes care of off mass shellness of couplings B^* or B_A^* with $B\rho$ channel [30] as indicated in eq. (5.9). We took the value of $g_+(0) = 0.29 \pm 0.04$ from a recent LCSR calculations [11]. And predict the value of $g_{B^*B\rho} = 2\,(1.26 \pm 0.17)\,/f_B\,\mathrm{GeV}^{-1}$ (cf. eq. (5.15)), which agrees very well with CQM calculation (5.16) and the value obtained by QCD sum rules $\left[\sqrt{2}\,(11)\ \mathrm{GeV}^{-1}\right]$. Also, we have predicted a relation between S-wave and D-wave couplings $g_{B_A^*B\rho^-} = 2.66 \times f_{B_A^*B\rho^-}\ \mathrm{GeV}^2$

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given in eq. (5.17). The estimate for $A_0(0) \simeq 0.320 \pm 0.044$ is consistant with the value given in eq. (5.27) and with the LCSR value 0.372 [11]. We summarize the form factors in eq. (5) and their values at $q^2 = 0$ in eq. (5.33).

A detailed analysis of form factors and helicity amplitudes is presented. Total decay rate is obtained which is $15.82\, |V_{ub}|^2$ ps^{-1} (cf. eq. 6.8), which results in predicting the CKM matrix element $|V_{ub}|$

$$|V_{ub}| = 3.24 \times 10^{-3}$$

by using CLEO branching ratio result [27], and

$$|V_{ub}| = 3.67 \times 10^{-3}$$

by using BaBar branching ratio result [28]. While using the branching ratio of recent study of CLEO [29], we obtain the value of $|V_{ub}|$

$$|V_{ub}| = 2.98 \times 10^{-3}\,.$$

Our results of $|V_{ub}|$ agree very well with the respective results of $|V_{ub}|$ of CLEO [27], and the recent results of BaBar [28], and CLEO [29]. The error shown in our result of $\Gamma(B \to \rho l \nu)$ in (6.8) is only 13.8%.

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A. Derivation of ward identities

We start with the relation

$$\langle V(k,\epsilon)\,| i\bar{u}\sigma^{\mu\nu}q_\nu\gamma_5 b|\, B(p)\rangle\, e^{-i(p-k)\cdot x} = -\,\langle V(k,\epsilon)\,|\partial_\nu\left(\bar{u}(x)\,\sigma^{\mu\nu}\gamma_5 b(x)\right)|\, B(p)\rangle\,. \quad \text{(A.1)}$$

We can replace ∂_ν by the covariant derivative D_ν to take into account the strong interaction of the the quark field. We can write the right hand side of eq. (A.1) as

$$-\langle V(k,\epsilon)|D_\nu\bar{u}(x)[-i\gamma^\nu\gamma^\mu + ig^{\mu\nu}]\gamma_5 b(x)|B(p)\rangle - \langle V(k,\epsilon)|\bar{u}(x)[i\gamma^\mu\gamma^\nu - ig^{\mu\nu}]\gamma_5 D_\nu b(x)|B(p)\rangle \quad \text{(A.2)}$$

Using Dirac equation

$$\not{D}b(x) = -im_b b(x), \qquad \bar{u}(x)\,\not{D} = im_u\bar{u}(x) \quad \text{(A.3)}$$

the relation (A.2) becomes

$$\begin{aligned}&(m_b - m_u)\langle V(k,\epsilon)|\bar{u}(x)\gamma^\mu\gamma_5 b(x)|B(p)\rangle - \\ &-i\langle V(k,\epsilon)|D^\mu\bar{u}(x)\gamma_5 b(x)|B(p)\rangle + i\langle V(k,\epsilon)|\bar{u}(x)\gamma_5 D^\mu b(x)|B(p)\rangle = \\ &\qquad = (m_b - m_u)\langle V(k,\epsilon)|\bar{u}\gamma^\mu\gamma_5 b|B(p)\rangle e^{-iq\cdot x} - i\langle V(k,\epsilon)|D^\mu(\bar{u}(x)\gamma_5 b(x))|B(p)+\rangle \\ &\qquad\quad + 2i\langle V(k,\epsilon)|\bar{u}(x)\gamma_5 D^\mu b(x)|B(p)\rangle\,. \end{aligned} \quad \text{(A.4)}$$

Using now the linear momentum commutation relation

$$\left[\hat{P}^\mu, O(x)\right] = -iD^\mu O(x)\,, \qquad \hat{P}^\mu_q = \int d^3x : q^\dagger(x) D^\mu q(x) :$$

and

$$\begin{aligned} \langle V(k,\epsilon)|\,\hat{P}^\mu &= k^\mu \,\langle V(k,\epsilon)| \\ \hat{P}^\mu |B(p)\rangle &= p^\mu |B(p)\rangle\,, \end{aligned} \tag{A.5}$$

the last two terms in eq. (A.4) become

$$-q^\mu \,\langle V(k,\epsilon)\,|\bar{u}\gamma_5 b|\, B(p)\rangle\, e^{-iq\cdot x} + 2\left\langle V(k,\epsilon)\,\left|\bar{u}\gamma_5 b p^\mu_b\right|\, B(p)\right\rangle e^{-iq\cdot x} \tag{A.6}$$

where in the last term we have used that $P^\mu_b\,|V(k,\epsilon)\rangle = 0$ as $V(k,\epsilon)$ does not contain the quark b. In the heavy quark effective theory m_b is taken to infinity in a way that p^μ_b/m is held fixed, but the four momentum of the light degree of freedom are neglected compared with m_b [51]. This enables us to identify p^μ_b with p^μ. Hence from eqs. (A.1), (A.4) and (A.6), we obtain the Ward identity (eq. (3.2)) $[2p - q = p \to k]$

$$\begin{aligned} \langle V(k,\epsilon)\,|i\bar{u}\sigma^{\mu\nu}q_\nu\gamma_5 b|\, B(p)\rangle &= (m_b - m_u)\,\langle V(k,\epsilon)\,|\bar{u}\gamma^\mu\gamma_5 b|\, B(p)\rangle + \\ &\quad + (p^\mu + k^\mu)\,\langle V(k,\epsilon)\,|\bar{u}\gamma_5 b|\, B(p)\rangle\,. \end{aligned} \tag{A.7}$$

Similarly,

$$\begin{aligned} \langle V(k,\epsilon)\,|i\bar{u}\sigma^{\mu\nu}q_\nu b|\, B(p)\rangle &= -\,(m_b - m_u)\,\langle V(k,\epsilon)\,|\bar{u}\gamma^\mu b|\, B(p)\rangle + \\ &\quad + (p^\mu + k^\mu)\,\langle V(k,\epsilon)\,|\bar{u}b|\, B(p)\rangle\,. \end{aligned} \tag{A.8}$$

However, the matrix element $\langle V(k,\epsilon)\,|\bar{u}b|\, B(p)\rangle$ vanishes due to parity consideration. Thus, we obtain the Ward Identity (3.1) given in the text.

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